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Colliding branes and formation of spacetime singularities in string theory

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ABSTRACT: Colliding branes without Z_2 symmetry and the formation of spacetime singularities in string theory are studied. After developing the general formulas to describe such events, we study a particular class of exact solutions first in the 5-dimensional effective theory, and then lift it to the 10-dimensional spacetime. In general, the 5-dimensional spacetime is singular, due to the mutual focus of the two colliding 3-branes. Non-singular cases also exist, but with the price that both of the colliding branes violate all the three energy conditions, weak, dominant, and strong. After lifted to 10 dimensions, we find that the spacetime remains singular, whenever it is singular in the 5-dimensional effective theory. In the cases where no singularities are formed after the collision, we find that the two 8-branes necessarily violate all the energy conditions.

KEYWORDS: Field Theories in Higher Dimensions, Brane Dynamics in Gauge Theories, Cosmology of Theories beyond the SM

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1 Introduction

Branes in string/M-Theory are fundamental constituents [1], and of particular relevance to cosmology [2, 3]. These substances can move freely in the bulk, collide, recoil, reconnect,

and whereby, among other possibilities, form a brane gas in the early universe [4], or create an ekpyrotic/cyclic universe [5]. Understanding these processes is fundamental to both string/M-Theory and their applications to cosmology [6].

Recently, Maeda and his collaborators numerically studied the collision of two branes in a five-dimensional bulk, and found that the formation of a spacelike singularity after the collision is generic [7] (See also [8]). This is an important result, as it implies that a low-energy description of colliding branes breaks down at some point, and a complete predictability is lost, without the complete theory of quantum gravity. Similar conclusions were obtained from the studies of two colliding orbifold branes [9]. However, lately it was argued that, from the point of view of the higher dimensional spacetime where the low effective action was derived, these singularities are very mild and can be easily regularised [10].

Lately, we constructed a class of exact solutions with two free parameters to the five-dimensional Einstein field equations, which represents the collision of two timelike 3-branes [11]. We found that, among other things, spacelike singularities generically develop after the collision, due to the mutual focus of the two branes. Non-singular spacetimes can be constructed only in the case where both of the two branes violate the energy conditions.

In this paper, we shall systematically study the collision of two timelike 8-branes without Z_2 symmetry in the framework of string theory. In particular, in section 2, starting with the Neveu-Schwarz/Neveu-Schwarz (NS-NS) sector in $(D+d)$ dimensions, $\hat{M}_{D+d} = M_D \times M_d$, we first obtain the D -dimensional effective theory in both the string frame and the Einstein frame, by toroidal compactification. To study the collision of two branes, we add brane actions to the D -dimensional effective action, and then derive the gravitational and matter field equations, including the ones on the two branes. In section 3, we apply these general formulas to the case where $D = 5 = d$ for a large class of spacetimes, and obtain the explicit field equations both outside the two branes and on the two branes. In section 4, we construct a class of exact solutions in the Einstein frame, in which the potential of the radion field on the two branes takes an exponential form, while the matter fields on the two branes are dust fluids. After identifying spacetime singularities both outside and on the branes, we are able to draw the corresponding Penrose diagrams for various cases. In sections 5, we study the local and global properties of these solutions in the 5-dimensional string frame, while in section 6 we first lift the solutions to 10 dimensions, and then study the local and global properties of these 10-dimensional solutions in detail. In section 7, we derive our main conclusions and present some remarks. There is also an appendix, in which we study a class of 10-dimensional spacetimes. In particular, we divide the Einstein tensor explicitly into three parts, one on each side of a colliding brane, and the other is on the brane. It is remarkable that the part on the brane can be written in the form of an anisotropic fluid.

2 The model

Let us consider the toroidal compactification of the NS-NS sector of the action in $(D+d)$ dimensions, $\hat{M}_{D+d} = M_D \times M_d$, where for string theory we have $D + d = 10$. Then, the

action takes the form [12],

$$S_{D+d} = -\frac{1}{2\kappa_{D+d}^2} \int d^{D+d}x \sqrt{|\hat{g}_{D+d}|} e^{-\hat{\Phi}} \left\{ \hat{R}_{D+d}[\hat{g}] + \hat{g}^{AB} \left(\hat{\nabla}_A \hat{\Phi} \right) \left(\hat{\nabla}_B \hat{\Phi} \right) - \frac{1}{12} \hat{H}^2 \right\}, \quad (2.1)$$

where in this paper we consider the $(D+d)$ -dimensional spacetimes described by the metric,

$$d\hat{s}_{D+d}^2 = \hat{g}_{AB} dx^A dx^B = \gamma_{ab}(x^c) dx^a dx^b + \hat{\phi}^2(x^c) \hat{\gamma}_{ij}(z^k) dz^i dz^j, \quad (2.2)$$

with $\gamma_{ab}(x^c)$ and $\hat{\phi}(x^c)$ depending only on the coordinates x^a of the spacetime M_D , and $\hat{\gamma}_{ij}(z^k)$ only on the internal coordinates z^k , where $a, b, c = 0, 1, 2, \dots, D-1$; $i, j, k = D, D+1, \dots, D+d-1$; and $A, B, C = 0, 1, 2, \dots, D+d-1$. Assuming that matter fields are all independent of z^k , one finds that the internal space M_d must be Ricci flat,

$$R[\hat{\gamma}] = 0. \quad (2.3)$$

For the purpose of the current work, it is sufficient to assume that M_d is a d -dimensional torus, $T^d = S^1 \times S^1 \times \dots \times S^1$. Then, we find that

$$\hat{R}_{D+d}[\hat{g}] = R_D[\gamma] + \frac{d(d-1)}{\hat{\phi}^2} \gamma^{ab} \left(\nabla_a \hat{\phi} \right) \left(\nabla_b \hat{\phi} \right) - \frac{2}{\hat{\phi}^d} \gamma^{ab} \left(\nabla_a \nabla_b \hat{\phi}^d \right). \quad (2.4)$$

Ignoring the dilaton $\hat{\Phi}$ and the form fields \hat{H} ,

$$\hat{\Phi} = 0 = \hat{H}, \quad (2.5)$$

we find that the integration of the action (2.1) over the internal space yields,

$$S_D = -\frac{1}{2\kappa_D^2} \int d^Dx \sqrt{|\gamma|} \hat{\phi}^d \left\{ R_D[\gamma] + \frac{d(d-1)}{\hat{\phi}^2} \gamma^{ab} \left(\nabla_a \hat{\phi} \right) \left(\nabla_b \hat{\phi} \right) \right\}, \quad (2.6)$$

where

$$\kappa_D^2 \equiv \frac{\kappa_{D+d}^2}{V_s}, \quad (2.7)$$

and V_s is defined as

$$V_s \equiv \int \sqrt{\tilde{\gamma}} d^d z. \quad (2.8)$$

For a string scale compactification, we have $V_s = \left(2\pi\sqrt{\alpha'} \right)^d$, where $(2\pi\alpha')$ is the inverse string tension.

After the conformal transformation,

$$g_{ab} = \hat{\phi}^{\frac{2d}{D-2}} \gamma_{ab}, \quad (2.9)$$

the D-dimensional effective action of eq. (2.6) can be cast in the minimally coupled form,

$$S_D^{(E)} = -\frac{1}{2\kappa_D^2} \int d^Dx \sqrt{|g_D|} \left\{ R_D[g] - \kappa_D^2 (\nabla\phi)^2 \right\}, \quad (2.10)$$

where

$$\phi \equiv \pm \left(\frac{(D+d-2)d}{\kappa_D^2(D-2)} \right)^{1/2} \ln(\hat{\phi}). \quad (2.11)$$

The action of eq. (2.6) is usually referred to as *the string frame*, and the one of eq. (2.10) as *the Einstein frame*. It should be noted that solutions related by this conformal transformation can have completely different physical and geometrical properties in the two frames. In particular, in one frame a solution can be singular, while in the other it can be totally free from any kind of singularities. A simple example is the flat FRW universe which is always conformally flat, $\gamma_{ab} = a^2(\tau)\eta_{ab}$. But the spacetime described by γ_{ab} usually has a big bang singularity, while the one described by η_{ab} is Minkowski, and does not have any kind of spacetime singularities.

To study the collision of two branes, we add the following brane actions to $S_D^{(E)}$ of eq. (2.10),

$$S_{D-1,m}^{(E,I)} = \int_{M_{D-1}^{(I)}} \sqrt{|g_{D-1}^{(I)}|} \left(\mathcal{L}_{D-1}^{(m,I)}(\psi) - V_{D-1}^{(I)}(\phi) \right) d^{D-1}\xi_{(I)}, \quad (2.12)$$

where $I = 1, 2$, $V_{D-1}^{(I)}(\phi)$ denotes the potential of the scalar field ϕ on the I-th brane, and $\xi_{(I)}^\mu$'s are the intrinsic coordinates of the I-th brane, where $\mu, \nu, \lambda = 0, 1, 2, \dots, D-2$. $\mathcal{L}_{D-1}^{(m,I)}(\psi)$ is the Lagrangian density of matter fields located on the I-th brane, denoted collectively by ψ . It should be noted that the above action does not include kinetic terms of the scalar field on the branes. This setup is quite similar to the Horava-Witten heterotic M-Theory on S^1/Z_2 [9, 13, 14], in which the two potentials $V_4^{(1)}(\phi)$ and $V_4^{(2)}(\phi)$ have opposite signs. It is also similar to the modulus stabilization mechanism of Goldberger and Wise [15], which has been lately applied to orbifold branes in string theory [16]. The two branes are localized on the surfaces,

$$\Phi_I(x^a) = 0, \quad (2.13)$$

or equivalently

$$x^a = x^a(\xi_{(I)}^\mu). \quad (2.14)$$

$g_{D-1}^{(I)}$ denotes the determinant of the reduced metric $g_{\mu\nu}^{(I)}$ of the I-th brane, defined as

$$g_{\mu\nu}^{(I)} \equiv g_{ab} e_{(\mu)}^{(I)a} e_{(\nu)}^{(I)b} \Big|_{M_{D-1}^{(I)}}, \quad (2.15)$$

where

$$e_{(\mu)}^{(I)a} \equiv \frac{\partial x^a}{\partial \xi_{(I)}^\mu} \Big|_{M_{D-1}^{(I)}}. \quad (2.16)$$

Then, the total action is given by,

$$S_{\text{total}}^{(E)} = S_D^{(E)} + \sum_{I=1}^2 S_{D-1,m}^{(E,I)}. \quad (2.17)$$

Variation of the total action (2.17) with respect to g_{ab} yields the D-dimensional gravitational field equations,

$$R_{ab} - \frac{1}{2}Rg_{ab} = \kappa_D^2 \left(T_{ab}^\phi + \sum_{I=1}^2 \left(T_{\mu\nu}^{(m,I)} + g_{\mu\nu}^{(I)} V_{D-1}^{(I)}(\phi) \right) e_a^{(I,\mu)} e_b^{(I,\nu)} \sqrt{\left| \frac{g_{D-1}^{(I)}}{g_D} \right|} \delta(\Phi_I) \right), \quad (2.18)$$

where

$$\begin{aligned} T_{ab}^\phi &= \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} (\nabla \phi)^2, \\ T_{\mu\nu}^{(m,I)} &= 2 \frac{\delta \mathcal{L}_{D-1}^{(m,I)}}{\delta g^{(I)}_{\mu\nu}} - g_{\mu\nu}^{(I)} \mathcal{L}_{D-1}^{(m,I)}, \end{aligned} \quad (2.19)$$

and $\nabla_a \left(\nabla_\mu^{(I)} \right)$ denotes the covariant derivative with respect to $g_{ab} \left(g_{\mu\nu}^{(I)} \right)$.

Variation of the total action with respect to ϕ , on the other hand, yields the Klein-Gordon field equations,

$$\square \phi = - \sum_{I=1}^2 \frac{\partial V_{D-1}^{(I)}(\phi)}{\partial \phi} \sqrt{\left| \frac{g_{D-1}^{(I)}}{g_D} \right|} \delta(\Phi_I), \quad (2.20)$$

where $\square \equiv g^{ab} \nabla_a \nabla_b$. We also have

$$\nabla_\nu^{(I)} T^{(m,I) \mu\nu} = 0. \quad (2.21)$$

Since we are mainly interested in collision of branes in string theory, in the rest of this paper we shall set $D = 5 = d$.

3 Colliding timelike 3-branes in the Einstein frame

We consider the 5-dimensional spacetime in the Einstein frame described by the metric,

$$ds_5^2 = g_{ab} dx^a dx^b = e^{2\sigma(t,y)} (dt^2 - dy^2) - e^{2\omega(t,y)} d\Sigma_0^2, \quad (3.1)$$

where $d\Sigma_0^2 \equiv (dx^2)^2 + (dx^3)^2 + (dx^4)^2$, and $x^0 = t$, $x^1 = y$. Then, the non-vanishing components of the Ricci tensor is given by

$$R_{tt} = - \{ 3\omega_{,tt} + \sigma_{,tt} + 3\omega_{,t}(\omega_{,t} - \sigma_{,t}) - \sigma_{,yy} - 3\omega_{,y}\sigma_{,y} \}, \quad (3.2)$$

$$R_{ty} = -3 \{ \omega_{,ty} + \omega_{,t}\omega_{,y} - \omega_{,t}\sigma_{,y} - \omega_{,y}\sigma_{,t} \}, \quad (3.3)$$

$$R_{yy} = - \{ 3\omega_{,yy} + \sigma_{,yy} + 3\omega_{,y}(\omega_{,y} - \sigma_{,y}) - \sigma_{,tt} - 3\omega_{,t}\sigma_{,t} \}, \quad (3.4)$$

$$R_{ij} = \delta_{ij} e^{2(\omega-\sigma)} \{ \omega_{,tt} + 3\omega_{,t}^2 - (\omega_{,yy} + 3\omega_{,y}^2) \}, \quad (3.5)$$

where now $i, j = 2, 3, 4$, and $\omega_{,t} \equiv \partial\omega/\partial t$, etc.

We assume that the two colliding 3-branes move along the hypersurfaces given, respectively, by

$$\begin{aligned} \Phi_1(t, y) &= t - ay = 0, \\ \Phi_2(t, y) &= t + by = 0, \end{aligned} \quad (3.6)$$

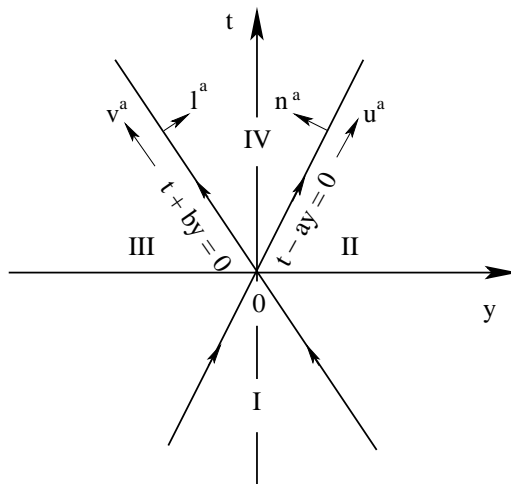


Figure 1. The five-dimensional spacetime in the (t, y) -plane for $a > 1, b > 1$. The two 3-branes are moving along the hypersurfaces, Σ_1 and Σ_2 , which are defined by eq. (3.10) in the text. The four regions, $I - IV$, are defined by eq. (3.8).

where a and b are two arbitrary constants, subjected to the constraints,

$$a^2 > 1, \quad b^2 > 1, \tag{3.7}$$

in order for the two hypersurfaces to be timelike. The two colliding branes divide the whole spacetime into four regions, $I - IV$, which are defined, respectively, as

$$\begin{aligned} \text{Region I} &\equiv \{x^a : \Phi_1 < 0, \Phi_2 < 0\}, \\ \text{Region II} &\equiv \{x^a : \Phi_1 > 0, \Phi_2 < 0\}, \\ \text{Region III} &\equiv \{x^a : \Phi_1 < 0, \Phi_2 > 0\}, \\ \text{Region IV} &\equiv \{x^a : \Phi_1 > 0, \Phi_2 > 0\}, \end{aligned} \tag{3.8}$$

as shown schematically in figure 1. In each of these regions, we define

$$F^A \equiv F(t, y)|_{\text{Region A}}, \tag{3.9}$$

where now $A = I, II, III, IV$.

We also define the two hypersurfaces Σ_1 and Σ_2 as,

$$\begin{aligned} \Sigma_1 &\equiv \{x^a : \Phi_1 = 0\}, \\ \Sigma_2 &\equiv \{x^a : \Phi_2 = 0\}. \end{aligned} \tag{3.10}$$

Then, it can be shown that the normal vectors to each of these two surfaces are given by

$$\begin{aligned} n_a &= N (\delta_a^t - a\delta_a^y), \\ l_a &= L (\delta_a^t + b\delta_a^y), \end{aligned} \tag{3.11}$$

where

$$\begin{aligned}
 F^{(I)} &\equiv F(t, y)|_{\Phi_I=0}, \\
 N &\equiv \frac{e^{\sigma^{(1)}}}{(a^2 - 1)^{1/2}}, \\
 L &\equiv \frac{e^{\sigma^{(2)}}}{(b^2 - 1)^{1/2}},
 \end{aligned}
 \tag{3.12}$$

with $F = \{\sigma, \omega, \phi\}$. We also introduce the two timelike vectors u_a and v_a via the relations,

$$\begin{aligned}
 u_a &= N (a\delta_a^t - \delta_a^y), \\
 v_a &= L (b\delta_a^t + \delta_a^y).
 \end{aligned}
 \tag{3.13}$$

It can be shown that these vectors have the following properties,

$$\begin{aligned}
 n_a n^a &= -1 = l_a l^a, \\
 u_a u^a &= +1 = v_a v^a, \\
 n_a u^a &= 0 = l_a v^a.
 \end{aligned}
 \tag{3.14}$$

In the following, we shall consider field equations, (2.18) and (2.20), in Regions $I - IV$ and along the hypersurfaces $\Sigma_{1,2}$, separately.

It should be noted that in the above setup, the two 3-branes do not have the Z_2 symmetry, in contrast to the setup of Horava-Witten in M theory [9, 13] and of Randall-Sundrum [17].

3.1 Field equations in regions $I - IV$

In these regions, the field equations of eqs. (2.18) and (2.20) take the form,

$$R_{ab}^A = \varphi_{,a}^A \varphi_{,b}^A, \tag{3.15}$$

$$\square^{(A)} \varphi^A = 0, \tag{3.16}$$

where $\varphi = \kappa_5 \phi$, and $\square^{(A)} \equiv g^{A ab} \nabla_a^{(A)} \nabla_b^{(A)}$, and $\nabla_a^{(A)}$ denotes the covariant derivative with respect to g_{ab}^A , and g_{ab}^A is the metric defined in Region A . From eq. (3.5) and the fact that $\varphi = \varphi(t, y)$, we find that

$$\omega = \frac{1}{3} \ln (f(t + y) + g(t - y)), \tag{3.17}$$

where $f(t + y)$ and $g(t - y)$ are arbitrary functions of their indicated arguments. Note that in writing eq. (3.17) we dropped the super indices A . In the following we shall adopt this convention, except for the case where confusions may raise. In the following we consider only the case where

$$f' g' \neq 0, \tag{3.18}$$

where a prime denotes the ordinary derivative with respect to the indicated argument. Then, introducing two new variables ξ_{\pm} via the relations,

$$\xi_{\pm}(t, y) \equiv f(t + y) \pm g(t - y), \tag{3.19}$$

we find that eq. (3.15) yields,

$$M_+ = \frac{1}{2}\xi_+(\varphi_+^2 + \varphi_-^2), \quad (3.20)$$

$$M_- = \xi_+\varphi_+\varphi_-, \quad (3.21)$$

and

$$M_{++} - M_{--} = -\frac{1}{2}(\varphi_+^2 - \varphi_-^2), \quad (3.22)$$

where $M_{\pm} \equiv \partial M / \partial \xi_{\pm}$, and

$$M(\xi_+, \xi_-) = \sigma + \frac{1}{3} \ln \xi_+ - \frac{1}{2} \ln(4f'g'). \quad (3.23)$$

On the other hand, eq. (3.16) can be cast in the form,

$$\varphi_{++} - \varphi_{--} + \frac{1}{\xi_+}\varphi_+ = 0. \quad (3.24)$$

It should be noted that eqs. (3.20)–(3.22) and (3.24) are not all independent. In fact, eq. (3.22) is the integrability condition of eqs. (3.20) and (3.21), and can be obtained from eqs. (3.20), (3.21) and (3.24). Therefore, in Regions $I - IV$, the field equations reduce to eqs. (3.20), (3.21) and (3.24).

To find solutions, one may first integrate eq. (3.24) to find φ , and then integrate eqs. (3.20) and (3.21) to find M . However, eq. (3.24) has infinite numbers of solutions, and the corresponding general solutions of M has not been worked out yet [18]. Once φ and M are known, the metric coefficients σ and ω are then given by

$$\begin{aligned} \sigma &= M - \frac{1}{3} \ln(f + g) + \frac{1}{2} \ln(4f'g'), \\ \omega &= \frac{1}{3} \ln(f + g). \end{aligned} \quad (3.25)$$

3.2 Field equations on the 3-branes

3.2.1 Field equations on the surface $\Phi_1 = 0$

Across the hypersurface $\Phi_1 = 0$, for any given C^0 function $F(t, y)$, it can be written as [19],

$$F(t, y) = F^+(t, y)H(\Phi_1) + F^-(t, y)[1 - H(\Phi_1)], \quad (3.26)$$

where F^+ (F^-) denotes the function $F(t, y)$ defined in the region $\Phi_1 > 0$ ($\Phi_1 < 0$), and $H(x)$ denotes the heaviside function, defined as

$$H(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0. \end{cases} \quad (3.27)$$

On the other hand, projecting $F_{,a}$ onto the n_a and u_a directions, we find

$$F_{,a} = F_u u_a - F_n n_a, \quad (3.28)$$

where

$$F_u \equiv u^a F_{,a}, \quad F_n \equiv n^a F_{,a}. \quad (3.29)$$

Since $[F_u]^- = 0$ due to the continuity of F across the branes, from the above expressions we find

$$[F_{,a}]^- = -[F_n]^- n_a, \quad (3.30)$$

where

$$[F_{,a}]^- \equiv \lim_{\Phi_1 \rightarrow 0^+} F_{,a}^+ - \lim_{\Phi_1 \rightarrow 0^-} F_{,a}^-. \quad (3.31)$$

Then, we find that

$$\begin{aligned} F_{,t} &= F_{,t}^+ H(\Phi_1) + F_{,t}^- [1 - H(\Phi_1)], \\ F_{,y} &= F_{,y}^+ H(\Phi_1) + F_{,y}^- [1 - H(\Phi_1)], \\ F_{,tt} &= F_{,tt}^+ H(\Phi_1) + F_{,tt}^- [1 - H(\Phi_1)] - N [F_n]^- \delta(\Phi_1), \\ F_{,ty} &= F_{,ty}^+ H(\Phi_1) + F_{,ty}^- [1 - H(\Phi_1)] + aN [F_n]^- \delta(\Phi_1), \\ F_{,yy} &= F_{,yy}^+ H(\Phi_1) + F_{,yy}^- [1 - H(\Phi_1)] - a^2 N [F_n]^- \delta(\Phi_1), \end{aligned} \quad (3.32)$$

where $\delta(\Phi_1)$ denotes the Dirac delta function. Then, we find that the Ricci tensor given by eqs. (3.2)–(3.5) can be cast in the form,

$$R_{ab} = R_{ab}^+ H(\Phi_1) + R_{ab}^- [1 - H(\Phi_1)] + R_{ab}^{Im} \delta(\Phi_1), \quad (3.33)$$

where R_{ab}^+ (R_{ab}^-) is the Ricci tensor calculated in the region $\Phi_1 > 0$ ($\Phi_1 < 0$), and R_{ab}^{Im} denotes the Ricci tensor calculated on the hypersurface $\Phi_1 = 0$, which has the following non-vanishing components,

$$\begin{aligned} R_{tt}^{Im} &= N \{ 3 [\omega_n]^- - (a^2 - 1) [\sigma_n]^- \}, \\ R_{ty}^{Im} &= -3aN [\omega_n]^-, \\ R_{yy}^{Im} &= N \{ 3a^2 [\omega_n]^- + (a^2 - 1) [\sigma_n]^- \}, \\ R_{ij}^{Im} &= N e^{2(\omega^{(1)} - \sigma^{(1)})} (a^2 - 1) [\omega_n]^- \delta_{ij}. \end{aligned} \quad (3.34)$$

On the hypersurface $\Phi_1 = 0$, the metric (3.1) reduces to

$$ds_5^2|_{\Phi_1=0} = g_{\mu\nu}^{(1)} d\xi_{(1)}^\mu d\xi_{(1)}^\nu = d\tau^2 - a_u^2(\tau) d\Sigma_0^2,$$

where $\xi_{(1)}^\mu \equiv \{\tau, x^2, x^3, x^4\}$, and

$$\begin{aligned} d\tau &\equiv \epsilon_\tau \left(\frac{a^2 - 1}{a^2} \right)^{1/2} e^{\sigma^{(1)}} dt, \\ a_u(\tau) &\equiv e^{\omega^{(1)}}, \end{aligned} \quad (3.35)$$

with $\epsilon_\tau = \pm 1$. Then, we find that

$$\begin{aligned} e_{(\tau)}^{(1)a} &\equiv \frac{\partial x^a}{\partial \tau} = \dot{t} \left(\delta_t^a + \frac{1}{a} \delta_y^a \right), \\ e_{(i)}^{(1)a} &\equiv \frac{\partial x^a}{\partial \xi_{(1)}^i} = \delta_i^a, \\ \sqrt{\left| \frac{g_4^{(1)}}{g_5} \right|} &= e^{-2\sigma^{(1)}}, \end{aligned} \tag{3.36}$$

where $i = 2, 3, 4$ and $\dot{t} \equiv dt/d\tau$. Then, the field equations of eq. (2.18) can be written as

$$[\omega_n]^- = \frac{\kappa_5^2 e^{-\sigma^{(1)}}}{3(a^2 - 1)^{1/2}} \left(\rho_m^{(1)} + V_4^{(1)} \right), \tag{3.37}$$

$$2[\omega_n]^- + [\sigma_n]^- = \frac{\kappa_5^2 e^{-\sigma^{(1)}}}{(a^2 - 1)^{1/2}} \left(V_4^{(1)} - p_m^{(1)} \right), \tag{3.38}$$

where in writing the above expressions we had assumed that $T_{\mu\nu}^{(m,1)}$ takes the form of a perfect fluid,

$$\begin{aligned} T_{\mu\nu}^{(m,1)} &\equiv \left(\rho_m^{(1)} + p_m^{(1)} \right) w_\mu^{(1)} w_\nu^{(1)} - p_m^{(1)} g_{\mu\nu}^{(1)}, \\ w_\mu^{(1)} &= \delta_\mu^\tau. \end{aligned} \tag{3.39}$$

Similarly, it can be shown that the Klein-Gordon equation (2.20) and the conservation law of the matter fields (2.21) on Σ_1 take, respectively, the forms,

$$[\phi_n]^- = -\frac{e^{-\sigma^{(1)}}}{(a^2 - 1)^{1/2}} \frac{\partial V_4^{(1)}(\phi)}{\partial \phi}, \tag{3.40}$$

$$\frac{d\rho_m^{(1)}}{d\tau} + 3H_u \left(\rho_m^{(1)} + p_m^{(1)} \right) = 0, \tag{3.41}$$

where $H_u \equiv \dot{a}_u/a_u$.

3.2.2 Field equations on the surface $\Phi_2 = 0$

Following a similar procedure as what we did in the last sub-section, one can show that the Ricci tensor across the brane $\Phi_2 = 0$ can be written as

$$R_{ab} = R_{ab}^+ H(\Phi_2) + R_{ab}^- [1 - H(\Phi_2)] + R_{ab}^{Im} \delta(\Phi_2), \tag{3.42}$$

where R_{ab}^+ (R_{ab}^-) now is the Ricci tensor calculated in the region $\Phi_2 > 0$ ($\Phi_2 < 0$), and R_{ab}^{Im} denotes the Ricci tensor calculated on the hypersurface $\Phi_2 = 0$, which has the following non-vanishing components,

$$\begin{aligned} R_{tt}^{Im} &= L \{ 3[\omega_l]^- - (b^2 - 1)[\sigma_l]^- \}, \\ R_{ty}^{Im} &= 3bL[\omega_l]^-, \\ R_{yy}^{Im} &= L \{ 3b^2[\omega_l]^- + (b^2 - 1)[\sigma_l]^- \}, \\ R_{ij}^{Im} &= L e^{2(\omega^{(2)} - \sigma^{(2)})} (b^2 - 1) [\omega_l]^- \delta_{ij}, \end{aligned} \tag{3.43}$$

where $\omega_l \equiv l^a \omega_{,a}$ etc. On the hypersurface $\Phi_2 = 0$, the metric (3.1) reduces to

$$ds_5^2|_{\Phi_2=0} = g_{\mu\nu}^{(2)} d\xi_{(2)}^\mu d\xi_{(2)}^\nu = d\eta^2 - a_v^2(\eta) d\Sigma_0^2,$$

where $\xi_{(2)}^\mu \equiv \{\eta, x^2, x^3, x^4\}$, and

$$\begin{aligned} d\eta &\equiv \epsilon_\eta \left(\frac{b^2 - 1}{b^2} \right)^{1/2} e^{\sigma^{(2)}} dt, \\ a_v(\eta) &\equiv e^{\omega^{(2)}}, \end{aligned} \tag{3.44}$$

with $\epsilon_\eta = \pm 1$. Then, we find that

$$\begin{aligned} e_{(\eta)}^{(2) a} &\equiv \frac{\partial x^a}{\partial \eta} = t^* \left(\delta_t^a - \frac{1}{b} \delta_y^a \right), \\ e_{(i)}^{(2) a} &\equiv \frac{\partial x^a}{\partial \xi_{(2)}^i} = \delta_i^a, \\ \sqrt{\left| \frac{g_4^{(2)}}{g_5} \right|} &= e^{-2\sigma^{(2)}}, \end{aligned} \tag{3.45}$$

where $t^* \equiv dt/d\eta$. Hence, the field equations of eq. (2.18) can be written as

$$[\omega_l]^- = \frac{\kappa_5^2 e^{-\sigma^{(2)}}}{3(b^2 - 1)^{1/2}} \left(\rho_m^{(2)} + V_4^{(2)} \right), \tag{3.46}$$

$$2[\omega_l]^- + [\sigma_l]^- = \frac{\kappa_5^2 e^{-\sigma^{(2)}}}{(b^2 - 1)^{1/2}} \left(V_4^{(2)} - p_m^{(2)} \right), \tag{3.47}$$

where in writing the above equations we had assumed that $T_{\mu\nu}^{(m,2)}$ takes the form,

$$\begin{aligned} T_{\mu\nu}^{(m,2)} &\equiv \left(\rho_m^{(2)} + p_m^{(2)} \right) w_\mu^{(2)} w_\nu^{(2)} - p_m^{(2)} g_{\mu\nu}^{(2)}, \\ w_\mu^{(2)} &= \delta_\mu^\eta. \end{aligned} \tag{3.48}$$

Similarly, it can be shown that the Klein-Gordon equation (2.20) and the conservation law of the matter fields (2.21) on Σ_2 take, respectively, the forms,

$$[\phi_l]^- = - \frac{e^{-\sigma^{(2)}}}{(b^2 - 1)^{1/2}} \frac{\partial V_4^{(2)}(\phi)}{\partial \phi}, \tag{3.49}$$

$$\frac{d\rho_m^{(2)}}{d\eta} + 3H_v \left(\rho_m^{(2)} + p_m^{(2)} \right) = 0, \tag{3.50}$$

where $H_v \equiv a_v^*/a_v$.

4 Particular solutions for colliding timelike 3-branes in the Einstein frame

Choosing the potentials $V_4^{(I)}(\phi)$ on the two branes as

$$V_4^{(I)}(\phi) = V_4^{(I,0)} e^{-\alpha\phi}, \tag{4.1}$$

where $V_4^{(I,0)}$'s and α are constants, and that the matter fields on each of the two branes are dust fluids, i.e.,

$$p_m^{(I)} = 0, \tag{4.2}$$

we find a class of solutions, which represents the collision of two timelike 3-branes and is given by

$$\begin{aligned} \sigma &= \left(\chi^2 - \frac{1}{3}\right) \ln(X_0 - X) + \sigma_0, \\ \omega &= \frac{1}{3} \ln(X_0 - X) + \omega_0, \\ \phi &= \frac{1}{\alpha} \ln(X_0 - X) + \phi_0, \end{aligned} \tag{4.3}$$

where $\chi \equiv \kappa_5/(\sqrt{2}\alpha)$, A_0 , σ_0 , ω_0 and ϕ_0 are arbitrary constants, and

$$X = b(t - ay) H(\Phi_1) + a(t + by) H(\Phi_2) = \begin{cases} (a + b)t, & IV, \\ a(t + by), & III, \\ b(t - ay), & II, \\ 0, & I. \end{cases} \tag{4.4}$$

The constants a and b are given by

$$\begin{aligned} b(a^2 - 1) &= \frac{3\kappa_5^2 V_4^{(1,0)}}{3\chi^2 + 1}, \\ a(b^2 - 1) &= -\frac{3\kappa_5^2 V_4^{(2,0)}}{3\chi^2 + 1}. \end{aligned} \tag{4.5}$$

When $\alpha = \pm\infty$, the solutions reduce to the ones studied previously [11]. So, in the rest of this paper we shall consider only the case where $\alpha \neq \pm\infty$. Without loss of generality, we can always set $\sigma_0 = \omega_0 = \phi_0 = 0$, and assume that

$$X_0 > 0. \tag{4.6}$$

It can be shown that the field equations, eqs. (3.15) and (3.16) [or eqs. (3.20), (3.21) and (3.24)], in Regions $I - IV$ are satisfied identically for the above solutions. To study the singular behavior of the spacetime in each of the four regions, we calculate the Ricci scalar, which in the present case is given by

$$R = \kappa_5^2 g^{ab} \phi_{,a} \phi_{,b} = \frac{\kappa_5^2 B}{\alpha^2 (X_0 - X)^{2(\chi^2 + 2/3)}}, \tag{4.7}$$

where X is given by eq. (4.4), and

$$B = \begin{cases} (a+b)^2, & IV, \\ -a^2(b^2-1), & III, \\ -b^2(a^2-1), & II, \\ 0, & I. \end{cases} \quad (4.8)$$

On the 3-brane located on $\Phi_1 = 0$, the reduced metric takes the form,

$$ds_5^2|_{\Sigma_1} = d\tau^2 - a_u^2(\tau)d^2\Sigma_0, \quad (4.9)$$

where

$$a_u(\tau) = \begin{cases} [\beta(\tau_s - \tau)]^{\frac{1}{3\chi^2+2}}, & \Phi_2 > 0, \\ X_0^{1/3}, & \Phi_2 < 0, \end{cases} \quad (4.10)$$

with

$$\begin{aligned} \Phi_2|_{\Phi_1=0} &= \frac{a+b}{a}t, \\ \beta &\equiv \frac{|a(a+b)|}{(a^2-1)^{1/2}} \left(\chi^2 + \frac{2}{3} \right), \\ \tau_s &\equiv \beta^{-1} X_0^{\chi^2 + \frac{2}{3}} \end{aligned} \quad (4.11)$$

Note that in writing the above expressions, we had chosen $\epsilon_\tau = \text{sign}(a+b)$. From eqs. (3.37) and (3.38), on the other hand, we find that

$$\rho_m^{(1)} = \frac{\rho_m^{(1,0)}}{X_0 - X^{(1)}(t)} = \begin{cases} [\beta(\tau_s - \tau)]^{-\frac{3}{3\chi^2+2}}, & \Phi_2 > 0, \\ X_0^{-1}, & \Phi_2 < 0, \end{cases} \quad (4.12)$$

where

$$\begin{aligned} \rho_m^{(1,0)} &\equiv \frac{b(a^2-1)}{\kappa_5^2} \left(\frac{2}{3} - \chi^2 \right), \\ X^{(1)}(t) &\equiv (a+b)tH(\Phi_2). \end{aligned} \quad (4.13)$$

From eqs. (4.3) and (4.4) we also find that

$$\phi^{(1)}(\tau) = \begin{cases} \frac{1}{\alpha(3\chi^2+2)} \ln[\beta(\tau_s - \tau)], & \Phi_2 > 0, \\ \frac{1}{\alpha} \ln X_0, & \Phi_2 < 0. \end{cases} \quad (4.14)$$

Similarly, on the 3-brane located on the hypersurface $\Phi_2 = 0$, the reduced metric takes the form,

$$ds_5^2|_{\Sigma_2} = d\eta^2 - a_v^2(\eta)d^2\Sigma_0, \quad (4.15)$$

where

$$a_v(\eta) = \begin{cases} [\gamma(\eta_s - \eta)]^{\frac{1}{3\chi^2+2}}, & \Phi_1 > 0, \\ X_0^{1/3}, & \Phi_1 < 0, \end{cases} \quad (4.16)$$

with $\epsilon_\eta = \text{sign}(a + b)$, and

$$\begin{aligned}\Phi_1|_{\Phi_2=0} &= \frac{a+b}{b}t, \\ \gamma &\equiv \frac{|b(a+b)|}{(b^2-1)^{1/2}} \left(\chi^2 + \frac{2}{3} \right), \\ \eta_s &\equiv \gamma^{-1} X_0^{\chi^2 + \frac{2}{3}}.\end{aligned}\tag{4.17}$$

The field equations (3.46) and (3.47), on the other hand, yield

$$\begin{aligned}\phi^{(2)}(\eta) &= \begin{cases} \frac{1}{\alpha(3\chi^2+2)} \ln [\gamma(\eta_s - \eta)], & \Phi_1 > 0, \\ \frac{1}{\alpha} \ln X_0, & \Phi_1 < 0, \end{cases}, \\ \rho_m^{(2)} = \frac{\rho_m^{(2,0)}}{X_0 - X^{(2)}(t)} &= \begin{cases} [\gamma(\eta_s - \eta)]^{-\frac{3}{3\chi^2+2}}, & \Phi_1 > 0, \\ X_0^{-1}, & \Phi_1 < 0, \end{cases}\end{aligned}\tag{4.18}$$

where

$$\begin{aligned}\rho_m^{(2,0)} &\equiv -\frac{a(b^2-1)}{\kappa_5^2} \left(\frac{2}{3} - \chi^2 \right), \\ X^{(2)}(t) &\equiv (a+b)tH(\Phi_1).\end{aligned}\tag{4.19}$$

It is interesting to note that when $\chi^2 = 2/3$, we have $\rho_m^{(I)} = 0$, ($I = 1, 2$), and the two 3-branes are supported only by the tensions $V_4^{(I)}(\phi)$, which are non-zero for any finite value of α [Recall the conditions (3.7)]. It is also remarkable to note that the presence of these two dust fluids is not essential to the singularity nature of the spacetime both in the bulk and on the branes. So, in the following we shall study the case with $\chi^2 = 2/3$ together with other cases.

To study the above solutions further, let us consider the following cases separately: (a) $a > 1, b > 1$; (b) $a > 1, b < -1$; (c) $a < -1, b > 1$; and (d) $a < -1, b < -1$.

4.1 $a > 1, b > 1$

In this case, from eq. (4.5) we find that

$$V_4^{(1)}(\phi) > 0, \quad V_4^{(2)}(\phi) < 0,\tag{4.20}$$

while eqs. (4.12) and (4.18) show that

$$\begin{aligned}\rho_m^{(1)} &= \begin{cases} \geq 0, & \chi^2 \leq 2/3, \\ < 0, & \chi^2 > 2/3, \end{cases}, \\ \rho_m^{(2)} &= \begin{cases} \leq 0, & \chi^2 \leq 2/3, \\ > 0, & \chi^2 > 2/3, \end{cases}.\end{aligned}\tag{4.21}$$

From eq. (4.7) we can also see that the spacetime is singular along the line $X_0 = (a+b)t$ in Region *IV*, the line $X_0 = a(t+by)$ in Region *III*, and the line $X_0 = b(t-ay)$ Region *II*, as shown by figure 2.

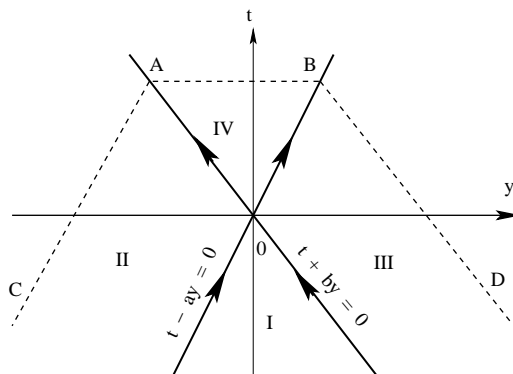


Figure 2. The five-dimensional spacetime in the (t, y) -plane for $a > 1, b > 1$. The two 3-branes are moving along the hypersurfaces, $\Sigma_1 : t - ay = 0$ and $\Sigma_2 : t + by = 0$. AB denotes the line $X_0 = (a + b)t$, AC the line $X_0 = b(t - ay)$, and BD the line $X_0 = a(t + by)$. The spacetime is singular along these lines. The four regions, $I - IV$, are defined by eq. (3.8).

Before the collision ($t < 0$), the scalar field is constant, $\phi^{(I)} = \phi_1 \equiv (1/\alpha) \ln(X_0)$, but both of the two potentials $V_4^{(1)}(\phi)$ and $V_4^{(2)}(\phi)$ are not zero, as are the dust energy densities $\rho_m^{(I)}$, except for $\chi^2 = 2/3$. In the case $\chi^2 = 2/3$, the dust fluids disappear and the two branes are supported only by tensions, denoted by the two constant potentials $V_4^{(1)}(\phi_1)$ and $V_4^{(2)}(\phi_1)$, which have the opposite signs, and are quite similar to the case of Randall-Sundrum (RS) branes [17], except for that in the RS model the two branes have Z_2 symmetry, while here we do not have. Before the collision, the spacetime on the two branes are flat, that is, the matter fields on the 3-brane do not curve the 3-branes. However, it does curve the spacetime outside the 3-branes. This is quite similar to the so-called self-tuning mechanism of brane worlds [20].

After the collision, the two 3-branes focus each other and finally a spacetime singularity is developed at, respectively, $\tau = \tau_s$ and $\eta = \eta_s$. The spacetime on the two branes is homogeneous and isotropic, and is described, respectively, by eqs. (4.9)–(4.10) and eqs. (4.15)–(4.16). The corresponding Penrose diagram is given by figure 3.

4.2 $a > 1, b < -1$

In this case, we find that

$$\begin{aligned}
 &V_4^{(1)}(\phi) < 0, \quad V_4^{(2)}(\phi) < 0, \\
 &\rho_m^{(I)} = \begin{cases} \geq 0, & \chi^2 \geq 2/3, \\ < 0, & \chi^2 < 2/3. \end{cases} \tag{4.22}
 \end{aligned}$$

Thus, unlike the last case, now both potentials $V_4^{(I)}(\phi)$ are negative, while the two dust energy densities always have the same sign.

To study the solutions further in this case, we shall consider the two subcases, $a > |b| > 1$ and $|b| > a > 1$, separately.

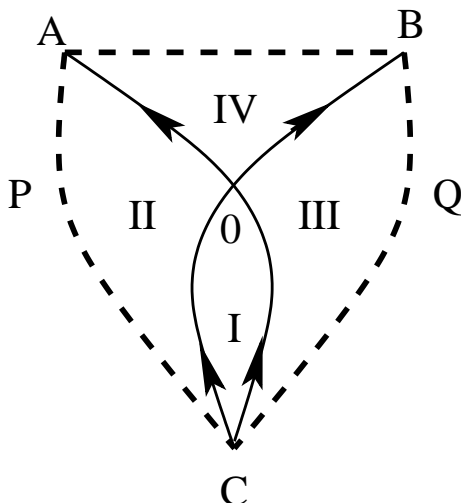


Figure 3. The Penrose diagram for $a > 1$, $b > 1$. The spacetime is singular along the straight line AB and the curved lines APC and BQC .

4.2.1 $a > -b > 1$

When $a > -b > 1$, we have

$$\begin{aligned}
 \Phi_1|_{\Phi_2=0} &= -\frac{a-|b|}{|b|}t = \begin{cases} < 0, & t > 0, \\ > 0, & t < 0, \end{cases} \\
 \Phi_2|_{\Phi_1=0} &= \frac{a-|b|}{a}t = \begin{cases} > 0, & t > 0, \\ < 0, & t < 0, \end{cases} \\
 X_0 - X &= \begin{cases} X_0 - (a-|b|)t, & IV, \\ X_0 - a(t-|b|y), & III, \\ X_0 + |b|(t-ay), & II, \\ 0, & I. \end{cases} \tag{4.23}
 \end{aligned}$$

Then, we find that the spacetime is singular along the line $X_0 = (a - |b|)t$ in Region IV , and the line $X_0 = a(t - |b|y)$ in Region III , as shown in figure 4.

Before the collision ($t < 0$), the scalar field $\phi^{(1)}$ is constant on the 3-brane located on the hypersurface $\Sigma_1 : t - ay = 0$, so does the dust energy density $\rho_m^{(1)}$. In contrast, both the scalar field $\phi^{(2)}$ and the dust energy density $\rho_m^{(2)}$ are time-dependent on the 3-brane located on $\Sigma_2 : t - |b|y = 0$, and the corresponding spacetime is described by eqs. (4.15) and (4.16) with $\eta \leq 0$. Note that along the hypersurface Σ_2 , we have $\Phi_1 > 0$ for $t < 0$, as shown by eq. (4.23).

After the collision, the 3-brane along Σ_2 transfers its energy to the one along Σ_1 , so that its energy density $\rho_m^{(2)}$ and potential $V_4^{(2)}(\phi)$, as well as the scalar field $\phi^{(2)}$, become constant, while the energy density $\rho_m^{(1)}$ and the scalar field $\phi^{(1)}$ become time-dependent. Because of the mutual focus of the two branes, a spacetime singularity is finally developed at $\tau = \tau_s$, denoted by the point B in figure 4. Afterwards, the spacetime becomes also singular along the line $X_0 = (a - |b|)t$ in Region IV and the line $X_0 = a(t - |b|y)$ in Region III .

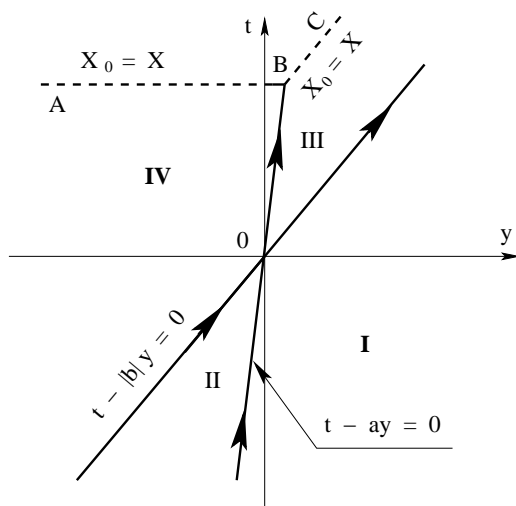


Figure 4. The five-dimensional spacetime in the (t, y) -plane for $a > -b > 1$. The two 3-branes are moving along the hypersurfaces, $\Sigma_1 : t - ay = 0$ and $\Sigma_2 : t - |b|y = 0$. The spacetime is singular along the line AB in Region IV and the line BC in Region III . The spacetime is also singular on the 3-brane at the point B where $\tau = \tau_s$. The four regions, $I - IV$, are defined by eq. (3.8).

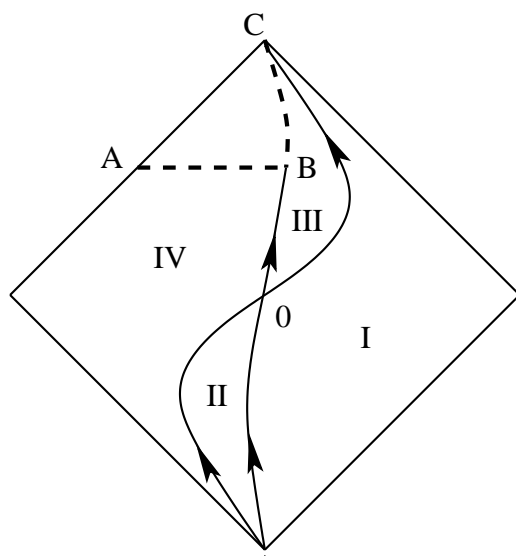


Figure 5. The Penrose diagram for $a > -b > 1$. The spacetime is singular along the lines AB and BC .

It is interesting to note that these singularities are always formed, regardless of the signs of $\rho_m^{(1)}$ and $\rho_m^{(2)}$. In fact, they are formed even when $\rho_m^{(1)}(\chi^2 = 2/3) = 0 = \rho_m^{(2)}(\chi^2 = 2/3)$, as can be seen from eqs. (4.7), (4.9) and (4.10). This is because the scalar field and the potentials $V_4^{(I)}(\phi)$ are still non-zero, and due the non-linear interaction of the scalar field itself, spacetime singularities are still formed. The corresponding Penrose diagram is given by figure 5.

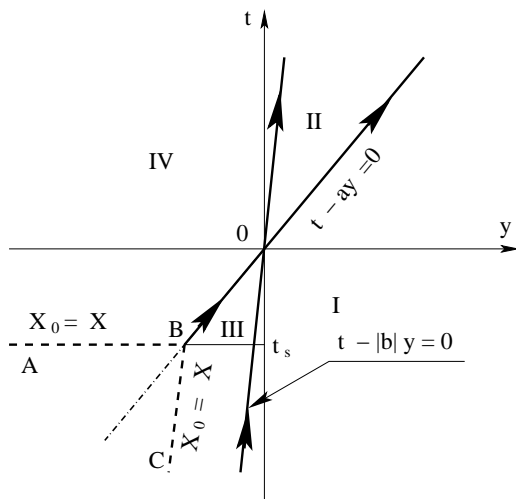


Figure 6. The five-dimensional spacetime in the (t, y) -plane for $-b > a > 1$. The two 3-branes are moving along the hypersurfaces, $\Sigma_1 : t - ay = 0$ and $\Sigma_2 : t - |b|y = 0$. The spacetime is singular along the line AB in Region IV and the line BC in Region III . The spacetime is also singular on the 3-brane at the point B .

4.2.2 $-b > a > 1$

When $-b > a > 1$, we have

$$\begin{aligned}
 \Phi_1|_{\Phi_2=0} &= -\frac{|b| - a}{|b|}t = \begin{cases} > 0, & t > 0, \\ < 0, & t < 0, \end{cases} \\
 \Phi_2|_{\Phi_1=0} &= -\frac{|b| - a}{a}t = \begin{cases} < 0, & t > 0, \\ > 0, & t < 0, \end{cases} \\
 X_0 - X &= \begin{cases} X_0 + (|b| - a)t, & IV, \\ X_0 - a(t - |b|y), & III, \\ X_0 + |b|(t - ay), & II, \\ 0, & I. \end{cases} \tag{4.24}
 \end{aligned}$$

Then, we find that the spacetime is singular along the line $X_0 = -(|b| - a)t$ in Region IV and the line $X_0 = (a - |b|)t$ in Region III , as shown in figure 6.

Unlike the last case, now the 3-brane on Σ_1 starts to expand at the singular point B where $\tau = \tau_s$, as shown in figure 6, and collides with the one on Σ_2 at the moment $\tau = 0$ ($t = 0$). After the collision, its energy density $\rho_m^{(1)}$ the scalar field $\phi^{(2)}$ and the dust energy density $\rho_m^{(2)}$ on Σ_2 become time-dependent, and the corresponding spacetime is described by eqs. (4.15) and (4.16) with $\eta \in (0, -\infty)$. The corresponding Penrose diagram is given by figure 7.

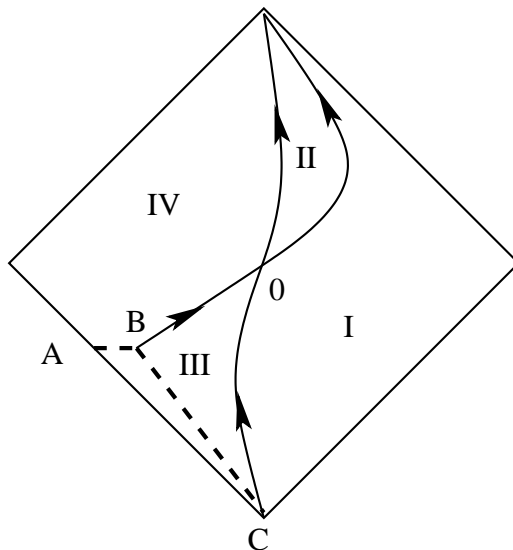


Figure 7. The Penrose diagram for $-b > a > 1$. The spacetime is singular along the lines AB and BC .

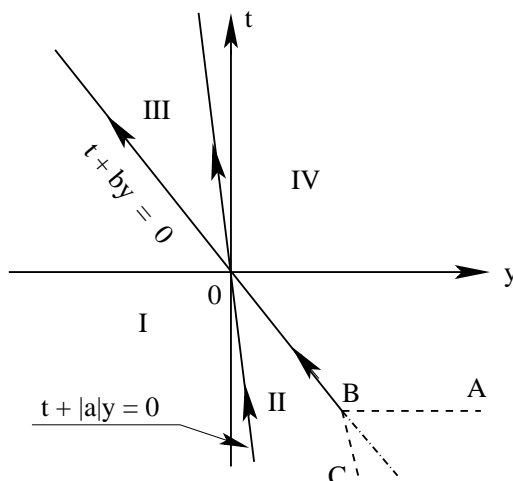


Figure 8. The five-dimensional spacetime in the (t, y) -plane for $-a > b > 1$. The two 3-branes are moving along the hypersurfaces, $\Sigma_1 : t + |a|y = 0$ and $\Sigma_2 : t + by = 0$. The spacetime is singular along the line AB in Region IV and the line BC in Region II . The spacetime is also singular on the 3-brane at the point B where $\eta = \eta_s$.

4.3 $a < -1, b > 1$

In this case, we find that

$$\begin{aligned}
 &V_4^{(I)}(\phi) > 0, \\
 &\rho_m^{(I)} = \begin{cases} \geq 0, & \chi^2 \leq 2/3, \\ < 0, & \chi^2 > 2/3, \end{cases} \tag{4.25}
 \end{aligned}$$

where $I = 1, 2$. Thus, in contrast to the last case, now both potentials $V_4^{(I)}(\phi)$ are positive, while the two dust energy densities always have the same sign.

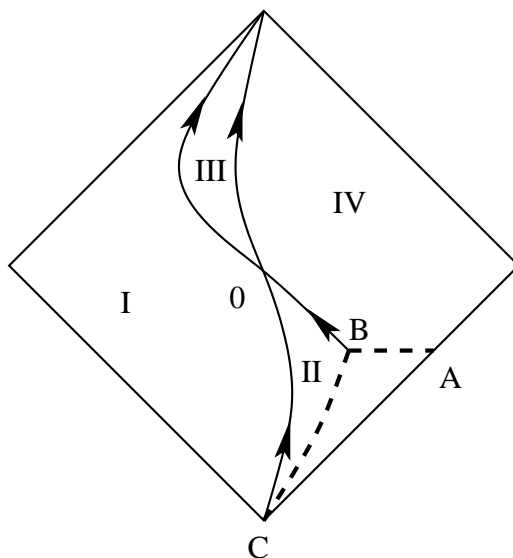


Figure 9. The Penrose diagram for $-a > b > 1$. The spacetime is singular along the lines AB and BC .

4.3.1 $-a > b > 1$

When $-a > b > 1$, we have

$$\begin{aligned}
 \Phi_1|_{\Phi_2=0} &= -\frac{|a|-b}{b}t = \begin{cases} < 0, & t > 0, \\ > 0, & t < 0, \end{cases} \\
 \Phi_2|_{\Phi_1=0} &= \frac{|a|-b}{|a|}t = \begin{cases} > 0, & t > 0, \\ < 0, & t < 0, \end{cases} \\
 X_0 - X &= \begin{cases} X_0 + (|a|-b)t, & IV, \\ X_0 + |a|(t+by), & III, \\ X_0 - b(t+|a|y), & II, \\ 0, & I. \end{cases} \tag{4.26}
 \end{aligned}$$

Then, the spacetime is singular along the line $X_0 = -(|a|-b)t$ in Region IV , and along the line $X_0 = b(t+|a|y)$ in Region II , as shown in figure 8. The corresponding Penrose diagram is given by figure 9.

In this case, we also have

$$\begin{aligned}
 \phi^{(1)}(\tau) &= \begin{cases} \frac{1}{\alpha(3\chi^2+2)} \ln [\beta(\tau_s - \tau)], & t > 0, \\ \frac{1}{\alpha} \ln X_0, & t < 0, \end{cases} \\
 \phi^{(2)}(\eta) &= \begin{cases} \frac{1}{\alpha} \ln X_0, & t > 0, \\ \frac{1}{\alpha(3\chi^2+2)} \ln [\gamma(\eta_s - \eta)], & t < 0, \end{cases} \\
 \rho_m^{(1)} &= \begin{cases} [\beta(\tau_s - \tau)]^{-\frac{3}{3\chi^2+2}}, & t > 0, \\ X_0^{-1}, & t < 0, \end{cases} \\
 \rho_m^{(2)} &= \begin{cases} X_0^{-1}, & t > 0, \\ [\gamma(\eta_s - \eta)]^{-\frac{3}{3\chi^2+2}}, & t < 0. \end{cases} \tag{4.27}
 \end{aligned}$$

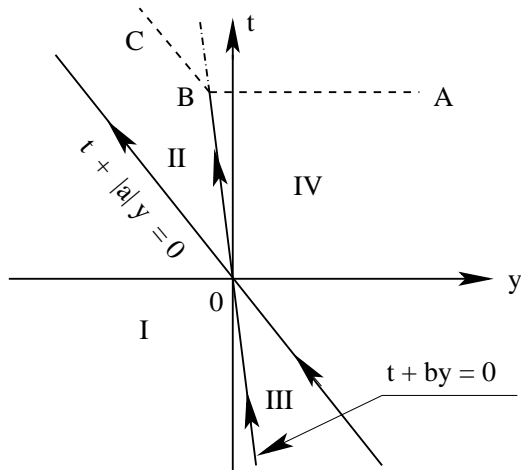


Figure 10. The five-dimensional spacetime in the (t, y) -plane for $b > -a > 1$. The two 3-branes are moving along the hypersurfaces, $\Sigma_1 : t + |a|y = 0$ and $\Sigma_2 : t + by = 0$. The spacetime is singular along the line AB in Region IV and the line BC in Region II . The spacetime is also singular on the 3-brane at the point B where $\eta = \eta_s$.

4.3.2 $b > -a > 1$

When $b > -a > 1$, we have

$$\begin{aligned}
 \Phi_1|_{\Phi_2=0} &= \frac{b - |a|}{b}t = \begin{cases} > 0, & t > 0, \\ < 0, & t < 0, \end{cases} \\
 \Phi_2|_{\Phi_1=0} &= -\frac{b - |a|}{|a|}t = \begin{cases} < 0, & t > 0, \\ > 0, & t < 0, \end{cases} \\
 X_0 - X &= \begin{cases} X_0 - (b - |a|)t, & IV, \\ X_0 + |a|(t + by), & III, \\ X_0 - b(t + |a|y), & II, \\ 0, & I. \end{cases} \tag{4.28}
 \end{aligned}$$

We also have

$$\begin{aligned}
 \phi^{(1)}(\tau) &= \begin{cases} \frac{1}{\alpha} \ln X_0, & t > 0, \\ \frac{1}{\alpha(3\chi^2+2)} \ln [\beta(\tau_s - \tau)], & t < 0, \end{cases} \\
 \phi^{(2)}(\eta) &= \begin{cases} \frac{1}{\alpha(3\chi^2+2)} \ln [\gamma(\eta_s - \eta)], & t > 0, \\ \frac{1}{\alpha} \ln X_0, & t < 0, \end{cases} \\
 \rho_m^{(1)} &= \begin{cases} X_0^{-1}, & t > 0, \\ [\beta(\tau_s - \tau)]^{-\frac{3}{3\chi^2+2}}, & t < 0, \end{cases} \\
 \rho_m^{(2)} &= \begin{cases} [\gamma(\eta_s - \eta)]^{-\frac{3}{3\chi^2+2}}, & t > 0, \\ X_0^{-1}, & t < 0. \end{cases} \tag{4.29}
 \end{aligned}$$

Then, the spacetime is singular along the line $X_0 = (b - |a|)t$ in Region IV , and along

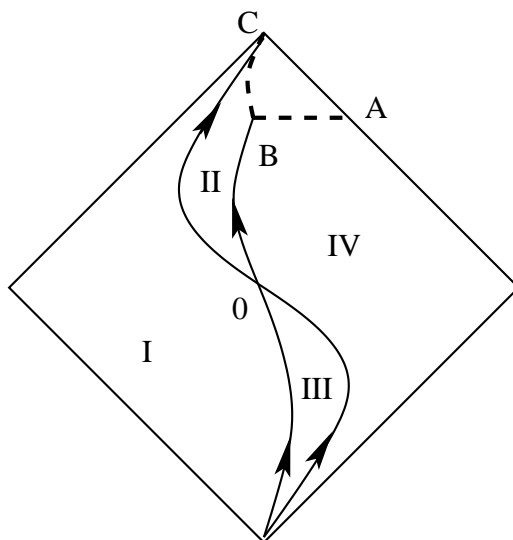


Figure 11. The Penrose diagram for $b > -a > 1$. The spacetime is singular along the lines AB and BC .

the line $X_0 = b(t + |a|y)$ in Region II , as shown in figure 10. The corresponding Penrose diagram is given by figure 11.

4.4 $a < -1, b < -1$

In this case, we have

$$\begin{aligned}
 V_4^{(1)}(\phi) &< 0, \quad V_4^{(2)}(\phi) > 0, \\
 \rho_m^{(1)} &= \begin{cases} \geq 0, & \chi^2 \geq 2/3, \\ < 0, & \chi^2 < 2/3, \end{cases} \\
 \rho_m^{(2)} &= \begin{cases} \geq 0, & \chi^2 \leq 2/3, \\ > 0, & \chi^2 > 2/3, \end{cases}
 \end{aligned} \tag{4.30}$$

and

$$\begin{aligned}
 \Phi_1|_{\Phi_2=0} &= \frac{|a| + |b|}{|b|}t = \begin{cases} > 0, & t > 0, \\ < 0, & t < 0, \end{cases} \\
 \Phi_2|_{\Phi_1=0} &= \frac{|a| + |b|}{|a|}t = \begin{cases} > 0, & t > 0, \\ < 0, & t < 0, \end{cases} \\
 X_0 - X &= \begin{cases} X_0 + (|a| + |b|)t, & IV, \\ X_0 + |a|(t - |b|y), & III, \\ X_0 + |b|(t + |a|y), & II, \\ 0, & I. \end{cases}
 \end{aligned} \tag{4.31}$$

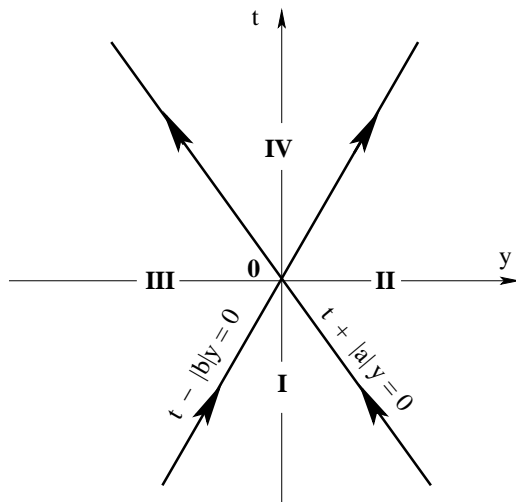


Figure 12. The five-dimensional spacetime in the (t, y) -plane for $a < -1$, $b < -1$. The two 3-branes are moving along the hypersurfaces, $\Sigma_1 : t + |a|y = 0$ and $\Sigma_2 : t - |b|y = 0$. The spacetime is free of any kind of spacetime singularities in the four regions, $I - IV$, as well as on the two 3-branes.

Then, we find that

$$\begin{aligned}
 \phi^{(1)}(\tau) &= \begin{cases} \frac{1}{\alpha(3\chi^2+2)} \ln [\beta (\tau_s - \tau)], & t > 0, \\ \frac{1}{\alpha} \ln X_0, & t < 0, \end{cases} \\
 \phi^{(2)}(\eta) &= \begin{cases} \frac{1}{\alpha(3\chi^2+2)} \ln [\gamma (\eta_s - \eta)], & t > 0, \\ \frac{1}{\alpha} \ln X_0, & t < 0, \end{cases} \\
 \rho_m^{(1)} &= \begin{cases} [\beta (\tau_s - \tau)]^{-\frac{3}{3\chi^2+2}}, & t > 0, \\ X_0^{-1}, & t < 0, \end{cases} \\
 \rho_m^{(2)} &= \begin{cases} [\gamma (\eta_s - \eta)]^{-\frac{3}{3\chi^2+2}}, & t > 0, \\ X_0^{-1}, & t < 0. \end{cases} \tag{4.32}
 \end{aligned}$$

Note that in the present case, after the collision $t > 0$, we have $\tau, \eta < 0$. Thus, in this case the spacetime is free of any kind singularity in all the four regions, as well as on the two branes, as shown in figure 12. The corresponding Penrose diagram is given by figure 13.

It is interesting to note that when $\chi^2 = 2/3$, the dust fluid on each of the two 3-branes disappears, and the branes are supported only by the tensions, where the brane along Σ_1 has a negative tension, while the one along Σ_2 has a positive tension. It is also interesting to note that, when $\chi^2 \neq 2/3$, both dust fluid are present, but they always have opposite signs, that is, if one satisfies the energy conditions [21], the other one must violate these conditions.

5 Colliding 3-branes in the 5-dimensional string frame

The spacetime singularity behavior in general can be quite different in the two frames, due to the conformal transformations of eq. (2.9), which are often singular. The 5-dimensional

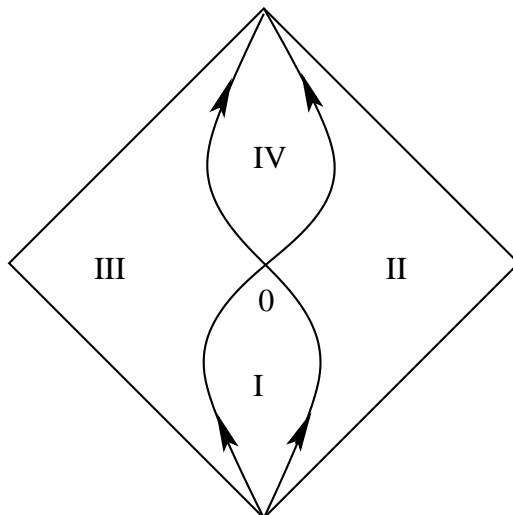


Figure 13. The Penrose diagram for $a < -1$, $b < -1$. The spacetime is non-singular in all the regions.

spacetime in the string frame is given by

$$d\hat{s}_5^2 \equiv \gamma_{ab} dx^a dx^b = e^{2\hat{\sigma}(t,y)} (dt^2 - dy^2) - e^{2\hat{\omega}(t,y)} d\Sigma_0^2, \tag{5.1}$$

where $d\Sigma_0^2$ is given in eq. (3.1), and

$$\begin{aligned} \hat{\sigma}(t,y) &\equiv \left(\chi^2 - \epsilon \sqrt{\frac{5}{12}} \chi - \frac{1}{3} \right) \ln(X_0 - X), \\ \hat{\omega}(t,y) &\equiv \left(\frac{1}{3} - \epsilon \sqrt{\frac{5}{12}} \chi \right) \ln(X_0 - X), \\ \hat{\phi}(t,y) &\equiv (X_0 - X)^{\epsilon \sqrt{\frac{3}{20}} \chi}, \end{aligned} \tag{5.2}$$

where $\epsilon = \pm 1$.

5.1 The spacetime singularities in regions I – IV

To study the spacetime singularities in Regions I – IV, let us consider the quantity,

$$\hat{\phi}_{,a} \hat{\phi}^{,a} = \frac{3\chi^2 B}{20(X_0 - X)^{\frac{4}{5} + \left(\sqrt{\frac{8}{15}} - \epsilon\sqrt{2}\chi\right)^2}}, \tag{5.3}$$

where B is given by eq. (4.8). Comparing the above expression with eq. (4.7), we find that the spacetime in Regions I – IV is singular in the string frame whenever it is singular in the Einstein frame, although the strength of the singularity is different, as can be seen clearly from the following expression,

$$20 \frac{\hat{\phi}_{,a} \hat{\phi}^{,a}}{R} = (X_0 - X)^{\epsilon \chi \sqrt{\frac{64}{15}}}, \tag{5.4}$$

In particular, if $\epsilon\alpha > 0$ the singularity in the Einstein frame is stronger, and if $\epsilon\alpha < 0$ it is the other way around.

5.2 The spacetime on the 3-brane $t = ay$

On the hypersurface $t = ay$, the metric (5.1) reduces to

$$d\hat{s}_5^2|_{t=ay} = d\hat{\tau}^2 - a_u^2(\hat{\tau}) d\Sigma_0^2, \quad (5.5)$$

where

$$a_u(\hat{\tau}) = \begin{cases} a_0(\hat{\tau}_s - \hat{\tau})^\Delta, & \Phi_2 > 0, \\ a_0\hat{\tau}_s^\Delta, & \Phi_2 < 0, \end{cases}$$

$$\hat{\phi}^{(1)}(\hat{\tau}) = \begin{cases} [\hat{\beta}(\hat{\tau}_s - \hat{\tau})]^{\epsilon\sqrt{\frac{3}{20}\frac{\chi}{\delta}}}, & \Phi_2 > 0, \\ (\hat{\beta}\hat{\tau}_s)^{\epsilon\sqrt{\frac{3}{20}\frac{\chi}{\delta}}}, & \Phi_2 < 0, \end{cases} \quad (5.6)$$

with

$$X_0 - X^{(1)} = \begin{cases} [\hat{\beta}(\hat{\tau}_s - \hat{\tau})]^{\frac{1}{\delta}}, & \Phi_2 > 0, \\ X_0, & \Phi_2 < 0, \end{cases}$$

$$\Phi_2|_{\Phi_1=0} = \frac{a+b}{a}t, \quad \hat{\beta} \equiv \frac{|a(a+b)|}{\sqrt{a^2-1}}\delta,$$

$$\hat{\tau}_s \equiv \hat{\beta}^{-1}X_0^\delta, \quad a_0 \equiv \hat{\beta}^\Delta.$$

$$\delta \equiv \left(\sqrt{\frac{5}{48}} - \epsilon\chi\right)^2 + \frac{9}{16} > 0,$$

$$\Delta \equiv \frac{1}{\delta} \left(\frac{1}{3} - \epsilon\chi\sqrt{\frac{5}{12}}\right). \quad (5.7)$$

Note that in writing the above expressions, we had chosen $\epsilon_{\hat{\tau}} = \text{sign}(a+b)$. To study the spacetime singularity on the brane, we calculate the Ricci scalar, which now is given by

$$R_u^{(4)\lambda}{}_{\lambda} = \frac{3\Delta(2-\Delta)}{2a_0(\hat{\tau}_s - \hat{\tau})^{\Delta+2}}, \quad (5.8)$$

where

$$\Delta + 2 = \frac{1}{\delta} \left[2 \left(\epsilon\chi - \sqrt{\frac{15}{64}} \right)^2 + \frac{115}{96} \right] > 0,$$

$$\Delta - 2 = -\frac{1}{\delta} \left[2 \left(\epsilon\chi - \sqrt{\frac{5}{48}} \right)^2 + \frac{19}{24} \right] < 0. \quad (5.9)$$

5.3 The spacetime on the 3-brane $t = -by$

Similarly, on the 3-brane located on the hypersurface $\Phi_2 = 0$, the metric (5.1) reduces to

$$d\hat{s}_5^2|_{t=-by} = d\hat{\eta}^2 - a_v^2(\hat{\eta}) d\Sigma_0^2, \quad (5.10)$$

where where

$$\begin{aligned}
 a_v(\hat{\eta}) &= \begin{cases} a_0 (\hat{\eta}_s - \hat{\eta})^\Delta, & \Phi_1 > 0, \\ a_0 \hat{\eta}_s^\Delta, & \Phi_1 < 0, \end{cases} \\
 \hat{\phi}^{(2)}(\hat{\eta}) &= \begin{cases} [\hat{\gamma} (\hat{\eta}_s - \hat{\eta})]^\epsilon \sqrt{\frac{3}{20} \frac{\chi}{\delta}}, & \Phi_1 > 0, \\ (\hat{\gamma} \hat{\eta}_s)^\epsilon \sqrt{\frac{3}{20} \frac{\chi}{\delta}}, & \Phi_1 < 0, \end{cases}
 \end{aligned} \tag{5.11}$$

with

$$\begin{aligned}
 X_0 - X^{(2)} &= \begin{cases} [\hat{\gamma} (\hat{\eta}_s - \hat{\eta})]^\frac{1}{\delta}, & \Phi_1 > 0, \\ X_0, & \Phi_1 < 0, \end{cases} \\
 \Phi_1|_{\Phi_2=0} &= \frac{a+b}{b}t, \quad \hat{\gamma} \equiv \frac{|a(a+b)|}{\sqrt{b^2-1}}\delta, \\
 \hat{\eta}_s &\equiv \hat{\gamma}^{-1}X_0^\delta,
 \end{aligned} \tag{5.12}$$

but now we have $a_0 \equiv \hat{\gamma}^\Delta$ and $\epsilon_{\hat{\eta}} = \text{sign}(a+b)$. For the metric (5.10), we also find that

$$R_{\nu \lambda}^{(4)} = \frac{3\Delta(2-\Delta)}{2a_0(\hat{\eta}_s - \hat{\eta})^{\Delta+2}}. \tag{5.13}$$

From eqs. (5.8) and (5.13) we can see that the spacetime on each of the branes is not singular when $\Delta = 0$ or $\chi = \epsilon\sqrt{\frac{4}{15}}$. As a matter of fact, in this case the spacetime on each of the two branes is flat. Thus, in the following we need to consider only the case $\chi \neq \epsilon\sqrt{\frac{4}{15}}$.

From eqs. (5.6)–(5.9) and eqs. (5.11)–(5.13), it can be shown that the spacetime singularities on each of the two branes are similar to these in the Einstein frame. For example, for the case $a > 1$, $b > 1$, it is singular at $\hat{t} = \hat{t}_s$ and $\hat{\eta} = \hat{\eta}_s$, which correspond to, respectively, the point *A* and *B* in figure 3. Similarly, the spacetime is free from any kind of singularities for the case $a < -1$, $b < -1$, and the corresponding Penrose diagram is also given by figure 13.

6 Colliding 8-branes in the 10-dimensional spacetimes

Lifting the metric to 10-dimensions, it is given by eq. (3.1), which can be cast in the form,

$$\begin{aligned}
 d\hat{s}_{10}^2 &\equiv \gamma_{ab}dx^a dx^b + \hat{\phi}^2(x^c) \hat{\gamma}_{ij}(z^k) dz^i dz^j \\
 &= e^{2\hat{\sigma}(t,y)} (dt^2 - dy^2) - e^{2\hat{\omega}(t,y)} d\Sigma_0^2 - \hat{\phi}^2(t,y) d\Sigma_z^2,
 \end{aligned} \tag{6.1}$$

where $\hat{\sigma}$, $\hat{\omega}$ and $\hat{\phi}$ are given by eq. (5.2), and $d\Sigma_z^2 \equiv \sum_{i,j=1}^5 \hat{\gamma}_{ij}(z^k) dz^i dz^j$. Then, it can be shown that the spacetime in Regions *I* – *IV* is vacuum,

$$R_{AB}^{(A)} = 0, \tag{6.2}$$

where $A = I, \dots, IV$, as it is expected. To study the singular behavior of the spacetime in these regions, we calculate the Kretschmann scalar, which in the present case is given by

$$I_{10} \equiv R_{ABCD}R^{ABCD} = \frac{B^2 I_{10}^{(0)}}{(X_0 - X) \left(2\chi - \epsilon\sqrt{\frac{5}{12}} \right)^2 + \frac{9}{4}}, \tag{6.3}$$

where B is given by eq. (4.8), and

$$I_{10}^{(0)} \equiv \frac{1}{45} \left[(720\chi^6 + 1287\chi^4 + 200\chi^2 + 40) - 312\epsilon\sqrt{\frac{5}{3}}\chi^3(2 + 3\chi^2) \right]. \quad (6.4)$$

It can be shown that $I_{10}^{(0)}$ is non-zero for any given χ . Then, comparing the expression of eq. (6.3) with eq. (4.7), we find that the lifted 10-dimensional spacetime has a similar singular behavior as that in the 5-dimensional spacetime in the Einstein frame. In particular, it is also singular on the hypersurface $X_0 - X = 0$.

On the hypersurface $t = ay$, the metric (6.1) reduces to

$$d\hat{s}_5^2|_{t=ay} = d\hat{\tau}^2 - a_u^2(\hat{\tau}) d\Sigma_0^2 - b_u^2(\hat{\tau}) d\Sigma_z^2, \quad (6.5)$$

where $a_u(\hat{\tau})$ and $b_u(\hat{\tau}) \equiv \hat{\phi}^{(1)}(\hat{\tau})$ are given by eqs. (5.6) and (5.7). On the 8-brane, the Einstein tensor has distribution given by eqs. (A.8) and (A.9). Inserting eq. (5.2) into eq. (5.7), and noticing that $\hat{\psi} \equiv \ln(\hat{\phi})$, we find

$$\begin{aligned} \hat{\rho}_u &= \frac{b(a^2 - 1)}{[X_0 - X^{(1)}(t)]^\mu}, \\ \hat{p}_u^Z &= -\frac{b(a^2 - 1)}{[X_0 - X^{(1)}(t)]^\mu} \left[\left(\chi - \epsilon\sqrt{\frac{4}{15}} \right)^2 + \frac{2}{5} \right], \\ \hat{p}_u^X &= -\frac{b(a^2 - 1)}{[X_0 - X^{(1)}(t)]^\mu} \left(\chi^2 + \frac{1}{3} \right), \end{aligned} \quad (6.6)$$

where $X^{(1)}(t)$ is given by eq. (5.7), and

$$\mu \equiv 2 \left(\chi - \epsilon\sqrt{\frac{5}{48}} \right)^2 + \frac{1}{8}. \quad (6.7)$$

Clearly, whenever $X_0 - X^{(1)}(t) = 0$, the spacetime on the 8-brane is singular.

On the hypersurface $t = -by$, the metric (6.1) reduces to

$$d\hat{s}_5^2|_{t=-by} = d\hat{\eta}^2 - a_v^2(\hat{\eta}) d\Sigma_0^2 - b_v^2(\hat{\eta}) d\Sigma_z^2, \quad (6.8)$$

where $a_v(\hat{\eta})$ and $b_v(\hat{\eta}) \equiv \hat{\phi}^{(2)}(\hat{\eta})$ are given by eqs. (5.11) and (5.12). On this 8-brane, the Einstein tensor has distribution given by eqs. (A.11) and (A.12), which in the present case yield,

$$\begin{aligned} \hat{\rho}_v &= \frac{a(b^2 - 1)}{[X_0 - X^{(2)}(t)]^\mu}, \\ \hat{p}_v^Z &= -\frac{a(b^2 - 1)}{[X_0 - X^{(2)}(t)]^\mu} \left[\left(\chi - \epsilon\sqrt{\frac{4}{15}} \right)^2 + \frac{2}{5} \right], \\ \hat{p}_v^X &= -\frac{a(b^2 - 1)}{[X_0 - X^{(2)}(t)]^\mu} \left(\chi^2 + \frac{1}{3} \right), \end{aligned} \quad (6.9)$$

where $X^{(2)}(t)$ is given by eq. (5.12). Thus, the spacetime on this 8-brane is also singular whenever $X_0 - X^{(2)}(t) = 0$.

When $a > 1$ and $b > 1$, from eqs. (6.6) and (6.9) it can be shown that both of the weak and dominant energy conditions [21] are satisfied by the matter fields on the two 8-branes, provided that

$$\begin{cases} -\frac{1}{\sqrt{15}} \leq \chi \leq \sqrt{\frac{2}{3}}, & \epsilon = +1, \\ -\sqrt{\frac{2}{3}} \leq \chi \leq \frac{1}{\sqrt{15}}, & \epsilon = -1, \end{cases} \quad (6.10)$$

but the strong energy condition is always violated. When $a > 1$ and $b < -1$, the matter field on the 8-brane $\Phi_1 = 0$ violates all the three energy conditions, while the one on the 8-brane $\Phi_2 = 0$ satisfies the weak and dominant energy conditions, provided that the conditions (6.10) holds, but violates the strong one. When $a < -1$ and $b > 1$, it is the other way around, that is, the matter field on the 8-brane $\Phi_1 = 0$ satisfies the weak and dominant energy conditions, provided that the conditions (6.10) holds, but violates the strong one, while the one on the 8-brane $\Phi_2 = 0$ violates all the three energy conditions. When $a < -1$ and $b < -1$, the matter fields on the two 8-branes all violate the three energy conditions. However, in all these four cases, the spacetime singular behavior is similar to the corresponding 5-dimensional cases in the Einstein frame. In particular, in the first three cases the spacetime in the four regions and on the 8-branes are always singular, and the corresponding Penrose diagrams are given, respectively, by figures 3, 5, 7, 9, and 11, but now each point in these figures now represents a 8-dimensional spatial space. In the last case, in which the matter fields on the two 8-branes violate all the energy conditions, the spacetime is free of any kind of spacetime singularities, either in Regions $I - IV$ or on the two 8-branes, and the corresponding Penrose diagram is given by figure 13. Therefore, all the above results seemingly indicate that violating the energy conditions is a necessary condition for spacetimes of colliding branes to be non-singular.

7 Conclusions

In this paper, we have first developed the general formulas to describe the collision of two timelike (D-1)-branes without Z_2 symmetry in a D-dimensional effective theory, obtained from the toroidal compactification of the Neveu-Schwarz/Neveu-Schwarz (NS-NS) sector in (D+d) dimensions. Applying the formulas to the case $D = 5 = d$ for a class of spacetimes, in section 3 we have obtained explicitly the field equations both outside and on the 3-branes in terms of distributions. In section 4, we have considered a class of exact solutions that represents the collision of two 3-branes in the Einstein frame, and studied their local and global properties in details. We have found, among other things, that the collision in general ends up with the formation of spacetime singularities, due to the mutual focus of the colliding branes, although non-singular spacetime also exist, with the price that both of the two branes violate all the energy conditions, weak, strong and dominant. Similar conclusions hold also in the 5-dimensional string frame. This has been done in section 5. In section 6, after lifted the solutions to 10-dimensional spacetimes, we have found that the corresponding solutions represent the collision of two timelike 8-branes without

Z_2 symmetry. In some cases the two 8-branes satisfy the weak and dominant energy conditions, while in other case, they do not. But, in all these cases the strong energy condition is always violated. The formation of spacetime singularities due to the mutual focus of the two colliding branes occurs in general, although the non-singular cases also exist with the price that both of the two branes violate all the three energy conditions. The spacetime singular behavior is similar in the 5-dimensional effective theory to that of 10-dimensional string theory.

In this paper, we have ignored the dilaton $\hat{\Phi}$ and the three-form field \hat{H}_{ABC} . It would be very interesting to see how these fields affect the formation of the spacetime singularities. In addition, it would also be very interesting to see what might happen if the branes are allowed to collide more than one time.

Acknowledgments

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A Gravitational field equations in the 10-dimensional bulk and on the 8-branes

For the metric,

$$d\hat{s}_{10}^2 = e^{2\hat{\sigma}(t,y)} (dt^2 - dy^2) - e^{2\hat{\omega}(t,y)} d\Sigma_0^2 - \hat{\phi}^2(t,y) d\Sigma_z^2, \quad (\text{A.1})$$

where

$$d\Sigma_0^2 \equiv \sum_{p=2}^4 (dx^p)^2, \quad d\Sigma_z^2 \equiv \sum_{i=1}^5 (dz^i)^2, \quad (\text{A.2})$$

the non-vanishing components of the Einstein tensor are given by,

$$\begin{aligned} G_{tt}^{(10)} &= 3\hat{\omega}_{,t}(\hat{\sigma}_{,t} + \hat{\omega}_{,t}) + 5\hat{\psi}_{,t}(\hat{\sigma}_{,t} + 3\hat{\omega}_{,t} + 2\hat{\psi}_{,t}) - 3\hat{\omega}_{,yy} - 5\hat{\psi}_{,yy} - 15\hat{\psi}_{,y}(\hat{\omega}_{,y} + \hat{\psi}_{,y}) \\ &\quad + \hat{\sigma}_{,y}(3\hat{\omega}_{,y} + 5\hat{\psi}_{,y}) - 6\hat{\omega}_{,y}^2, \\ G_{ty}^{(10)} &= -3\hat{\omega}_{,ty} - 5\hat{\psi}_{,ty} + 3(\hat{\sigma}_{,t}\hat{\omega}_{,y} + \hat{\sigma}_{,y}\hat{\omega}_{,t} - \hat{\omega}_{,t}\hat{\omega}_{,y}) + 5(\hat{\sigma}_{,t}\hat{\psi}_{,y} + \hat{\sigma}_{,y}\hat{\psi}_{,t} - \hat{\psi}_{,t}\hat{\psi}_{,y}), \\ G_{yy}^{(10)} &= -3\hat{\omega}_{,tt} - 5\hat{\psi}_{,tt} - 15\hat{\psi}_{,t}(\hat{\omega}_{,t} + \hat{\psi}_{,t}) + \hat{\sigma}_{,t}(3\hat{\omega}_{,t} + 5\hat{\psi}_{,t}) - 6\hat{\omega}_{,t}^2 + 3\hat{\omega}_{,y}(\hat{\sigma}_{,y} + \hat{\omega}_{,y}) \\ &\quad + 5\hat{\psi}_{,y}(\hat{\sigma}_{,y} + 3\hat{\omega}_{,y} + 2\hat{\psi}_{,y}), \\ G_{pq}^{(10)} &= \delta_{pq}e^{2(\hat{\omega}-\hat{\sigma})} \left[\hat{\sigma}_{,yy} + 2\hat{\omega}_{,yy} + 5\hat{\psi}_{,yy} + 5\hat{\psi}_{,y}(2\hat{\omega}_{,y} + 3\hat{\psi}_{,y}) + 3\hat{\omega}_{,y}^2 \right. \\ &\quad \left. - (\hat{\sigma}_{,tt} + 2\hat{\omega}_{,tt} + 5\hat{\psi}_{,tt} + 3\hat{\omega}_{,t}^2 + 5\hat{\psi}_{,t}(2\hat{\omega}_{,t} + 3\hat{\psi}_{,t})) \right], \\ G_{ij}^{(10)} &= \delta_{ij}e^{2(\hat{\psi}-\hat{\sigma})} \left[\hat{\sigma}_{,yy} + 3\hat{\omega}_{,yy} + 4\hat{\psi}_{,yy} + 2\hat{\psi}_{,y}(6\hat{\omega}_{,y} + 5\hat{\psi}_{,y}) + 6\hat{\omega}_{,y}^2 \right. \\ &\quad \left. - (\hat{\sigma}_{,tt} + 3\hat{\omega}_{,tt} + 4\hat{\psi}_{,tt} - 10\hat{\psi}_{,t}^2 + 6\hat{\omega}_{,t}(\hat{\omega}_{,t} + 2\hat{\psi}_{,t})) \right], \end{aligned} \quad (\text{A.3})$$

where $p, q = 2, 3, 4$ and $i, j = 1, \dots, 5$, and $\hat{\psi} \equiv \ln(\hat{\phi})$.

A.1 Field equations on the hypersurface $\Phi_1 = 0$

Following section 3.2.1, it can be shown that the derivatives of any given function $F(t, y)$, which is C^0 across the hypersurface $\Phi_1 = 0$ and at least C^2 in the regions $\Phi_1 > 0$ and $\Phi_1 < 0$, are given by eq. (3.32) but now with N being replaced by \hat{N} , and n_a and u_a by, respectively, \hat{n}_a and \hat{u}_a , where

$$\begin{aligned}\hat{n}_a &= \hat{N} (\delta_a^t - a\delta_a^y), \\ \hat{u}_a &= \hat{N} (a\delta_a^t - \delta_a^y), \\ \hat{N} &\equiv \frac{e^{\hat{\sigma}^{(1)}}}{(a^2 - 1)^{1/2}}.\end{aligned}\tag{A.4}$$

Hence, eq. (A.3) can be cast in the form,

$$G_{ab}^{(10)} = G_{ab}^{(10)+} H(\Phi_1) + G_{ab}^{(10)-} [1 - H(\Phi_1)] + G_{ab}^{(10)Im} \delta(\Phi_1),\tag{A.5}$$

where $G_{ab}^{(10)+}$ ($G_{ab}^{(10)-}$) is the Einstein tensor calculated in the region $\Phi_1 > 0$ ($\Phi_1 < 0$), and $G_{ab}^{(10)Im}$ denotes the distribution of the Einstein tensor on the hypersurface $\Phi_1 = 0$, which has the following non-vanishing components,

$$\begin{aligned}G_{tt}^{(10)Im} &= a^2 \hat{N} \left(3 [\hat{\omega}_n]^- + 5 [\hat{\psi}_n]^- \right), \\ G_{ty}^{(10)Im} &= -a \hat{N} \left(3 [\hat{\omega}_n]^- + 5 [\hat{\psi}_n]^- \right), \\ G_{yy}^{(10)Im} &= \hat{N} \left(3 [\hat{\omega}_n]^- + 5 [\hat{\psi}_n]^- \right), \\ G_{pq}^{(10)Im} &= -\delta_{pq} \hat{N}^{-1} e^{2\hat{\omega}^{(1)}} \left([\hat{\sigma}_n]^- + 2 [\hat{\omega}_n]^- + 5 [\hat{\psi}_n]^- \right), \\ G_{ij}^{(10)Im} &= -\delta_{ij} \hat{N}^{-1} e^{2\hat{\psi}^{(1)}} \left([\hat{\sigma}_n]^- + 3 [\hat{\omega}_n]^- + 4 [\hat{\psi}_n]^- \right).\end{aligned}\tag{A.6}$$

Introducing the unit vectors,

$$X_a^{(p)} = e^{\hat{\omega}^{(1)}} \delta_a^p, \quad Z_a^{(i)} = e^{\hat{\psi}^{(1)}} \delta_a^i,\tag{A.7}$$

we find that eq. (A.6) can be cast in the form,

$$G_{ab}^{(10)Im} = \kappa_{10}^2 \left(\hat{\rho}_u \hat{u}_a \hat{u}_b + \hat{p}_u^X \sum_{p=2}^4 X_a^{(p)} X_b^{(p)} + \hat{p}_u^Z \sum_{i=1}^5 Z_a^{(i)} Z_b^{(i)} \right),\tag{A.8}$$

where

$$\begin{aligned}\hat{\rho}_u &= \frac{1}{\hat{N} \kappa_{10}^2} \left(3 [\hat{\omega}_n]^- + 5 [\hat{\psi}_n]^- \right), \\ \hat{p}_u^X &= \frac{1}{\hat{N} \kappa_{10}^2} \left([\hat{\sigma}_n]^- + 2 [\hat{\omega}_n]^- + 5 [\hat{\psi}_n]^- \right), \\ \hat{p}_u^Z &= \frac{1}{\hat{N} \kappa_{10}^2} \left([\hat{\sigma}_n]^- + 3 [\hat{\omega}_n]^- + 4 [\hat{\psi}_n]^- \right).\end{aligned}\tag{A.9}$$

A.2 Field equations on the hypersurface $\Phi_2 = 0$

Similarly, it can be shown that, crossing the hypersurface $\Phi_2 = 0$, eq. (A.3) can be cast in the form,

$$G_{ab}^{(10)} = G_{ab}^{(10)+} H(\Phi_2) + G_{ab}^{(10)-} [1 - H(\Phi_2)] + G_{ab}^{(10)Im} \delta(\Phi_2), \quad (\text{A.10})$$

but now $G_{ab}^{(10)+}$ ($G_{ab}^{(10)-}$) is the Einstein tensor calculated in the region $\Phi_2 > 0$ ($\Phi_2 < 0$), and $G_{ab}^{(10)Im}$ denotes the distribution of the Einstein tensor on the hypersurface $\Phi_2 = 0$, which can be written in the form,

$$G_{ab}^{(10)Im} = \kappa_{10}^2 \left(\hat{\rho}_v \hat{v}_a \hat{v}_b + \hat{p}_v^X \sum_{p=2}^4 X_a^{(p)} X_b^{(p)} + \hat{p}_v^Z \sum_{i=1}^5 Z_a^{(i)} Z_b^{(i)} \right), \quad (\text{A.11})$$

where

$$\begin{aligned} \hat{\rho}_v &= \frac{1}{\hat{L}\kappa_{10}^2} \left(3[\hat{\omega}_l]^- + 5[\hat{\psi}_l]^- \right), \\ \hat{p}_v^X &= \frac{1}{\hat{L}\kappa_{10}^2} \left([\hat{\sigma}_l]^- + 2[\hat{\omega}_l]^- + 5[\hat{\psi}_l]^- \right), \\ \hat{p}_v^Z &= \frac{1}{\hat{L}\kappa_{10}^2} \left([\hat{\sigma}_l]^- + 3[\hat{\omega}_l]^- + 4[\hat{\psi}_l]^- \right), \end{aligned} \quad (\text{A.12})$$

and

$$\begin{aligned} X_a^{(p)} &= e^{\hat{\omega}^{(2)}} \delta_a^p, & Z_a^{(i)} &= e^{\hat{\psi}^{(2)}} \delta_a^i, \\ \hat{l}_a &= \hat{L} (\delta_a^t + b\delta_a^y), & \hat{v}_a &= \hat{L} (b\delta_a^t + \delta_a^y), \\ \hat{L} &\equiv \frac{e^{\hat{\sigma}^{(2)}}}{(b^2 - 1)^{1/2}}. \end{aligned} \quad (\text{A.13})$$

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